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The phenomenon of runaway electrons in partially ionized non-ideal plasma

T S Ramazanov and K M Turekhanova

IETP, Al Farabi Kazakh National University, Tole Bi 96a, 050012 Almaty, Kazakhstan

E-mail: kunduz@physics.kz

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Abstract
The effect of runaway electrons in partially ionized hydrogen plasma is investigated on the basis of pseudopotential models. The conditions of runaway electrons were determined. Dependences of an electron free path on the plasma density and coupling parameter were obtained. It is shown that if the quantum-mechanical and screening effects in non-ideal partially ionized plasma are taken into consideration, the collision frequency curve for electrons has maxima and free path curves for electrons have minima.

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1. Introduction

The investigation into the phenomenon of runaway electrons is of fundamental and practical interest, because the superthermal electrons determine velocities of ionization and excitation of plasma neutral components [1–3]. In tokamaks, plasma electrons run away at generation above a critical energy due to the decrease of the Coulomb collision frequency with the increase of electron energy. In this connection, it is necessary to analyze the probability of runaway electrons in a system by studying physical properties of non-ideal plasma and to do numerical simulations to investigate this problem [4, 5].

In this work we consider the dense partially ionized hydrogen plasma; the number density is in the range $n_{\text{tot}} = n_e + n_i + n_a = 10^{20}$ to$10^{24}\,\text{cm}^{-3}$, where n_e, n_i, n_a are the densities of electrons, ions and atoms, respectively. The temperature domain is $T = (10^4\text{–}5 \times 10^6)\,\text{K}$. Dimensionless variables are used to describe the system: coupling parameter $\Gamma = e^2/ak_BT$. Here k_B is the Boltzmann constant, e is the electron charge, $a = \left(3/4\pi n_e\right)^{1/3}$ is the average distance between the particles and r_D is the Debye length, $r_S = a/a_B$ is the density parameter, where a_B is the Bohr radius.

2. Interaction models

In the present work, for a description of the interaction of charged particles in classical partially ionized plasma we used pseudopotential, taking into account three-particle correlation in a system [6]. And for a description of the interaction of charged particles in semiclassical dense partially ionized plasma, the effective pseudopotential, taking into consideration the quantum-mechanical and screening effects, was used [7],

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{\sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D}} \left(\frac{\mathrm{e}^{-Br}}{r} - \frac{\mathrm{e}^{-Ar}}{r} \right) \tag{1}$$

where $A^2 = \left(1 + \sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2}\right)/\left(2\lambda_{\alpha\beta}^2\right)$, $B^2 = \left(1 - \sqrt{1 - 4\lambda_{\alpha\beta}^2/r_D^2}\right)/\left(2\lambda_{\alpha\beta}^2\right)$, $Z_\alpha e$, $Z_\beta e$ are the electric charges of α and β of particles, $\lambda_{\alpha\beta} = \hbar/\sqrt{2\pi m_{\alpha\beta} k_B T}$ is the thermal de Broglie wavelength; $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass of α–β pair; $r_D = \left(k_B T/(4\pi e^2 \sum_j n_j Z_j^2)\right)^{1/2}$ is the Debye radius. This pseudopotential is valid when $\lambda_{\alpha\beta} < r_D/2$. The effective potential (1) was obtained on the basis of the micropotential proposed by Deutsch *et al* in [8, 9].

The influence of atoms on the effect of electron runaway in the partially ionized plasma increases with the decrease of free electron number density. It is known that the interaction between a charge and an atom in plasma is basically caused by the effects of polarization and is of short-range character. In this work, we used the screening version of the Buckingham potential as the potential of charge–atom interaction in partially ionized non-ideal plasma [10].

Also, as the potential of charge–atom interaction in partially ionized dense semiclassical plasma we used the polarization potential taking into account quantum-mechanical and screening effects [11]:

$$\Phi_{ea}(r) = -\frac{e^2\alpha_D}{2r^4\left(1 - 4\lambda_{\alpha\beta}^2/r_D^2\right)} \left(\mathrm{e}^{-Br}(1 + Br) - \mathrm{e}^{-Ar}(1 + Ar)\right)^2, \tag{2}$$

where α_D is the polarizability of atom.

The chemical model of plasma was used for the calculation of the composition of real partially ionized plasma. In this model plasma consists of all possible kinds of plasma particles. For hydrogen plasma these include electrons, ions and atoms. As is known, interaction of electrons and ions with atoms in dense plasma results in the decrease of the ionization potential. Determining the ionization ratio $\alpha = n_e^*/n_e$ as the ratio of free electrons' number to full number of electrons, we can write the Saha equation in this form [12]:

$$\frac{1 - \alpha}{\alpha^2} = n_e \lambda_{\alpha\beta}^3 \exp[(I - \Delta I)/k_B T] \tag{3}$$

where $I \approx 13.6$ eV is the ionization potential for hydrogen. $\Delta I = \Delta\mu$ is the lowering of the ionization potential, which is determined on the basis of the pseudopotential model of particle's interaction. The Saha equation is solved numerically.

3. Analysis

By runaway electrons we understand that electrons are accelerated in plasma due to some external electric field. Electrons, therefore, may undergo continuous acceleration. Such transition is called the runaway effect since electrons behave as if they are avoiding collisions. It happens due to the electron friction force that decreases with the increase in electron velocity

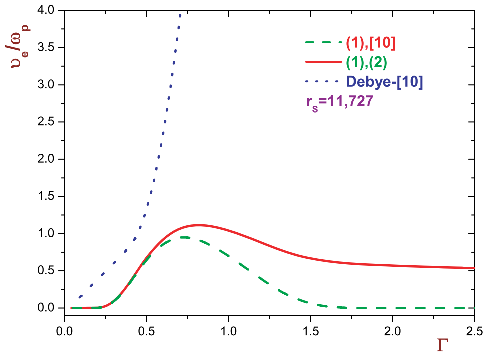


Figure 1. Collision frequency for electrons calculated on the basis of pseudopotential models of partially ionized semiclassical non-ideal plasma at $r_S = 11.727$.

See endnote 2

[13]. In the case of partially ionized plasma, the probability of runaway electrons is determined by their collision frequency calculated as [14]

$$\nu_e = \nu_{ei} + \nu_{ee} + \nu_{ea}, \qquad \nu_{e\beta} = n_\beta \sigma_{tr}^{e\beta} \upsilon_e, \tag{4}$$

where υ_e is the velocity of electrons and $\sigma_{tr}^{e\beta}$ is the transport cross section of a particle. In this work the transport cross section is calculated on the basis of phase functions [15]:

$$\sigma_{tr}^{e\beta} = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (l+1) \sin\left(\delta_{l+1}^{e\beta} - \delta_l^{e\beta}\right). \tag{5}$$

Here the wave number k is related to the kinetic energy E of relative motion via $k^2 = 2m_{\alpha\beta}E/\hbar^2$, the phase shifts $\delta_l^{e\beta}$ are obtained from the solution of the Calogero equation with the interaction potential:

$$\frac{\mathrm{d}\delta_l^{e\beta}(r)}{\mathrm{d}r} = -\frac{1}{k}\Phi(r)\left[\cos\delta_l^{e\beta}(r) j_l(kr) - \sin\delta_l^{e\beta}(r) n_l(kr)\right]^2. \tag{6}$$

$$\delta_l^{e\beta}(0) = 0, \qquad \delta_l^{e\beta} = \lim_{r\to\infty} \delta_l^{e\beta}(r),$$

where $j_l(kr)$ and $n_l(kr)$ are the known Ricatti–Bessel functions.

The collision frequency of electrons on the basis of effective pseudopotential models of semiclassical plasma as a function of the coupling parameter is presented in figure 1. When quantum-mechanical and screening effects are taken into consideration, particle frequency decreases with the increase of the coupling parameter. Calculation results for electron free path $l = \upsilon_e/\nu_e = 1/n\sigma_{tr}$ are presented in figure 2 as a function of the plasma coupling parameter for partially ionized hydrogen plasma. It is shown that the free path calculated within the pseudopotential model for classical non-ideal plasma is higher than the corresponding data for the Debye potential. It is well known that the free path decreases at higher particle density and cross section. There is a minimum at the free path curve within the pseudopotential model as the coupling parameter increases. It could be related to the increased role of high-order correlation effects, which leads to the decrease of cross section.

The critical electric field (Dreicer field) that determines the condition of electrons runaway in the system for fully ionized plasma, is defined by the formula [16]:

$$E_{cr} = \frac{4\pi e^3 n_e}{k_B T}\lambda_C \approx 2.6 \times 10^{-13} \frac{n_e}{k_B T_e}\lambda_C. \tag{7}$$

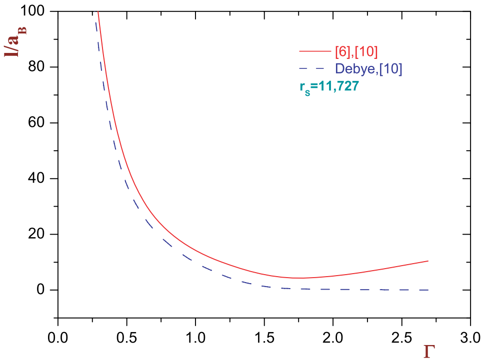
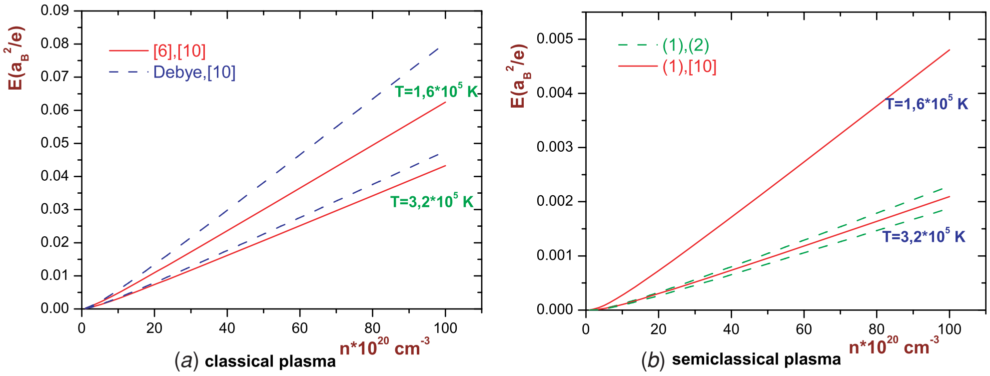


Figure 2. Electrons free path calculated on the basis of pseudopotential models for partially ionized hydrogen classical non-ideal plasma.



(*a*) **classical plasma** (*b*) **semiclassical plasma**

Figure 3. Critical electric field calculated on the basis of pseudopotential models for partially ionized hydrogen dense plasma.

Here E_{cr} is given in V cm $^{-1}$, n_e is given in cm^{-3} and $k_B T_e$ is given in eV. λ_C is the Coulomb logarithm determined using particle interaction potential [17].

Calculations of the critical electric field for partially ionized hydrogen plasma are presented in figure 3. The results are compared with those obtained with the Debye potential. The difference in results is explained by the consideration of the long-range many-particle screening effects. At the regime of weak non-ideality, i.e. the decrease of plasma density and temperature in the system, the results become close to each other.

4. Conclusions

Electron collision frequency decreases with the increase of the coupling plasma parameter if the quantum-mechanical and screening effects are taken into consideration. When higher order correlation effects are taken into account, the free path of electrons increases with the increase of the plasma coupling parameter in partially ionized hydrogen plasma compared to those calculated within the Debye approximation. The condition of runaway electrons in classical and semiclassical partially ionized plasma is intensified in the regime of dense plasma, i.e. the critical electric field calculated on the basis of the pseudopotential models increases with the increase of plasma density and with the decrease of plasma temperature. Consequently, the probability of runaway electrons in dense plasma is more than the same in rarefied plasma. This happens due to several factors: decrease of the effective impact parameter of scattering, and decrease of the collision frequency in non-ideal plasma [18] and formation of some ordered structures in dense plasma.

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